TRANSITION FROM DROPLET FLOW TO STREAM FLOW IN A LIQUID

V. F. Dunskii and N. V. Nikitin

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The transition from droplet flow to stream flow is examined theoretically for flow of liquid from a vertical capillary and runoff from the edge of an inclined plane under the influence of the force of gravity, and also for spraying of a liquid from a rotating perforated drum and a smooth disk. The formulas proposed agree satisfactorily with experiment.

In liquid atomization the dimensions of the droplets formed vary over wide ranges, but in certain liquid fractionation processes occurring under conditions of laminar flow with low flow rates droplets of uniform dimension are formed. Such "monodispersed" processes, significant in scientific studies, are of importance in technology as well (formation of powders from melts, spray drying, pesticide spraying).

Monodispersed fractionation of a liquid is accomplished in outflow from orifices and capillaries, runoff from a slit or edge, or throwoff from the edge of a rotating disk or perforated drum.

We will consider the flow of a nonwetting liquid from a vertical capillary under the effect of the force of gravity (Fig. 1a). At a low column height H the liquid does not flow from the capillary; a suspended drop is formed at its lower end, and the liquid weight is in equilibrium with the surface-tension force. The equilibrium condition is

$$2\pi R\sigma \geqslant (4\pi R^3/3 + \pi R^2 H)\rho z \tag{1}$$

where R is the capillary radius, ρ and σ are the density and surface tension of the liquid, and $z = g$ is the acceleration of gravity.

As H is increased the meniscus height h increases, then equilibrium is disrupted, and the liquid begins to move. Due to the interaction of surface tension and gravity the liquid flows in the form of discrete identical droplets (Fig. 1b, c) whose diameter is determined by the equality of the forces:

$$d_0 = (12 R\sigma/\rho z)^{1/3} \tag{2}$$

In order that the droplets formed be identical, the capillary radius must not exceed a critical value R_{max}, which is obtained from Eq. (1) at $H = 0$:

$$R_{max} = (3\sigma/2\rho z)^{1/2} \tag{3}$$

The validity of Eqs. (1)-(3) and the identical nature of the drops formed may be proved by simple experiment.

Equation (2) is valid for low liquid expenditure Q. With increase in Q the drop-formation frequency increases and in a certain range of Q values a crisis develops – the process is reorganized: as Q increases, together with the identical "basic" drops there are formed in the intervals between basic drops an increasing number of finer "satellite drops," then connection between adjacent drops ceases to be discontinuous, and the transition from drop formation to stream flow occurs; the liquid exits from the capillary in the form of a continuous stream.

For further increase in flow rate Q a second crisis develops – the transition from laminar to turbulent flow.

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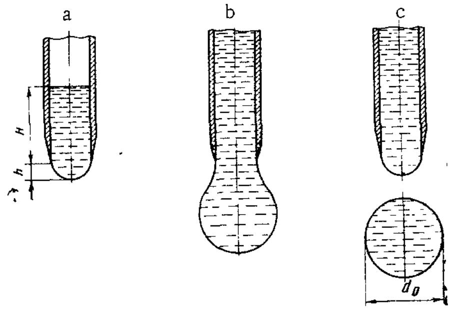

Fig. 1

We will consider here the conditions for onset of the first crisis, the transition from drop formation to stream flow. We shall consider the changes which increase in liquid flow rate Q produces in the droplet-formation process.

At low flow rates the liquid kinetic energy need not be considered, and drop formation can be considered the result of equality of two forces: the surface-tension force $2\pi R \sigma$ and the gravitational force $(\pi d^3/6)\rho z$. Equation (2) was obtained on the assumption of equality of these forces.

For increased flow rate Q the pressure within the forming drop produced by kinetic energy of the liquid cannot be neglected. The force acting on the drop as a result of braking of the liquid entering it is, on the average, equal to $\pi R^2(\rho u^2/2)$, where $u = Q/\pi R^2$ is the mean liquid outflow velocity from the capillary.

The balance of forces in drop formation with consideration of the effect of flow rate has the form

$$2\pi R\sigma = \frac{\pi d^3}{6}\rho z + \frac{\rho Q^2}{2\pi R^2} \qquad (4)$$

whence

$$d = \left(\frac{12R\sigma}{\rho z} - \frac{3Q^2}{\pi^2 R^2}\right)^{1/3} = d_0\left(1 - \frac{\rho Q^2}{4\pi^2 R^3 \sigma}\right)^{1/3} \qquad (5)$$

As is evident from Eq. (5), with growth in Q decrease in drop diameter d occurs.

With the drop-formation mechanism considered here the drop diameter cannot become less than the capillary diameter 2R. Hence, d = 2R may be taken as a limiting condition for onset of the drop-formation–stream-flow transition crisis. From Eq. (4) at d = 2R we obtain the limiting expression for critical flow rate:

$$Q_c = 2\pi K_1\left(\frac{R^3\sigma}{\rho}\right)^{1/2}\left[1 - \left(\frac{R}{R_{\max}}\right)^2\right]^{1/2} \qquad (6)$$

where K_1 is a coefficient considering the degree to which the real process approximates the given limit.

Another approach to determination of the critical flow rate Q_c is as follows. Each drop is formed in the course of a certain time τ, which is dependent on the drop diameter d and flow rate Q. After breakaway from the capillary the drop undergoes acceleration. This motion must occur sufficiently rapidly for a place to be freed for the next drop being formed, i.e., over a time period τ the broken-away drop must traverse a distance $S \geq d$. If $S < d$, then the drops must blend into one another. The condition $S = d$ may be taken as a limit for the critical regime.

The formation time for a drop of diameter d is

$$\tau = \pi d^3 / 6Q \qquad (7)$$

The distance traversed by a drop in falling from a state of rest under the influence of gravity (neglecting air resistance in view of the low velocity) is

$$S = z\tau^2 / 2 \qquad (8)$$

Taking the limiting condition S = d, we obtain from Eqs. (7), (8)

$$Q_c = \frac{\pi K_2}{6}\left(\frac{d^5}{2}\right)^{1/2} \qquad (9)$$

where d is determined from Eq. (2).

As in Eq. (6), the coefficient K_2 considers the degree of approximation of the real process to the given limit.

We will now consider the flow of a nonwetting liquid from a capillary under the influence of centrifugal force. A radial orifice in the wall of a drum rotating about its own axis will act as the capillary. The liquid is supplied to the drum center, flows along its inner surface under the action of the centrifugal force, fills the radial orifice, and is expelled outward in the form of drops.

TABLE 1

Liquid	Density ρ, g/cm^3	Viscosity η, g/ cm/sec	Surface tension σ, g/sec^2
Water	1.00	0.01	73
Glucose solution	1.20	0.086	104
Mercury	13.6	0.015	476
Transformer oil	0.892	0.194	33.2
Mineral oil	0.870	1.30	30.5
Lubricating oil	0.897	2.36	29.5
Glycerine	1.30	14.95	63.4
Diesel fuel	0.892	0.025	30.6

With a small quantity of liquid within the capillary, at its outer end a convex liquid meniscus is formed. In contrast to the case considered above, with its stationary capillary, forces produced by air friction act on this meniscus. The liquid in the meniscus must move in response to these forces. In the first approximation we will neglect this motion. Then Eqs. (2)-(9) are applicable to the rotating capillary with $z = r\omega^2$, i.e., with replacement of the gravitational acceleration g by the centrifugal acceleration $r\omega^2$, where r is the radius of the outer surface of the drum (whose wall thickness $h \ll r$), and ω is the angular velocity of its rotation. Then to determine Q_c by the first method Eq. (6) remains valid, but $R_{max} = (3\sigma/2\rho r\omega^2)^{1/2}$. To determine Q_c by the second method we use Eq. (9) in the form

$$Q_c = \frac{\pi}{6} K_2 \left(\frac{r\omega^2 d^5}{2}\right)^{1/2} \tag{10}$$

at

$$d = (12 R \sigma / \rho r \omega^2)^{1/3} \tag{11}$$

We will now turn to the formation of identical droplets with the surface well-wetted by the liquid. We will consider the runoff of a wetting liquid from the lower edge of an inclined plane under the influence of gravity.

If the surface is wetted by the liquid and there is no liquid supply (from the reservoir) then at the lower edge there will be formed a stationary liquid cylinder of radius a, whose weight $\pi a^2 l \rho z$ balances the surface-tension force $2l\sigma$ (l being the length of the edge).

The equilibrium condition is

$$\pi a^2 l \rho z \leqslant 2 l \sigma \tag{12}$$

whence the maximum radius of the pendant cylinder at equilibrium is

$$a = (2\sigma / \pi \rho z)^{1/2} \tag{13}$$

Here $z = g$, the acceleration of gravity.

If liquid is supplied to the plane from a reservoir the entire system goes into motion – equilibrium is disrupted, and the excess liquid forms drops which break away from the pendant cylinder and fall downward.

Drops form in those locations where the pendant liquid cylinder most easily loses its stability under the action of random perturbations. These locations are spaced at a distance λ, which may be approximately determined from the expression proposed in [1] for decay of a liquid cylinder:

$$\lambda = 9 a [1 + (4.5 \eta^2 / a \rho \sigma)^{1/2}] \tag{14}$$

At each of these instability points at small flow rates drops are formed as a result of interaction of the force of gravity $(\pi d_0^3/6)\rho z$ and the force of surface tension $\pi d_0 \sigma$.

The drop diameter will be

$$d_0 = C \left(\frac{\sigma}{\rho z}\right)^{1/2} \tag{15}$$

where C = const and $z = g$.

Equation (15) is valid for small flow rates Q. With increase in Q, as in the case of a capillary, the drop-formation frequency increases, and drop flow is replaced by stream flow.

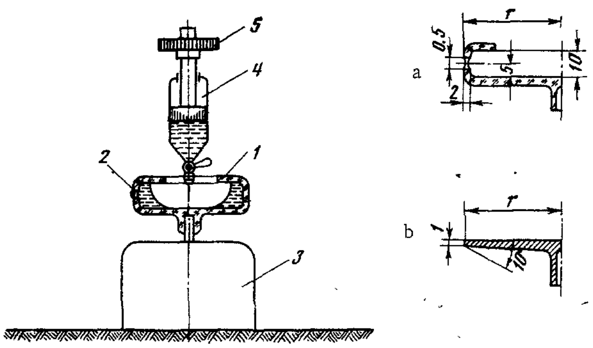

Fig. 2

For the first method of determining critical liquid flow rate Q_c, using the same considerations as in the case of the capillary, we arrive at an analogous force-balance equation for drop formation:

$$2\pi a\sigma = \frac{\pi d^3}{6}\rho z + \frac{\rho Q_1^2}{2\pi a^2} \tag{16}$$

Here instead of the capillary radius R we have the radius of the pendant cylinder a, determined approximately by Eq. (11); $Q_1 = (\lambda / l)$, Q is the liquid flow from one drop-formation point. Further evaluations analogous to those performed above for the capillary lead to the condition $d = 2a$, defining the transition to stream flow. From Eq. (16) at $d = 2a$ we find the expression for critical flow rate:

$$Q_c = 4.8K_1 \frac{l}{\lambda}\left(\frac{a^3\sigma}{\rho}\right)^{1/2} \tag{17}$$

To determine Q_c by the second method we use Eq. (9), taking $z = g$, $d = c(\sigma/\rho\, g)^{1/2}$, and introducing the factor l/λ

$$Q_c = \frac{\pi}{6} K_2 \frac{l}{\lambda}\left(\frac{gd^5}{2}\right)^{1/2} \tag{18}$$

We will now consider the drop-formation process of a wetting liquid on a rotating disk. Instead of the lower edge of the inclined plane we consider the edge of the rotating disk, to whose center is supplied a liquid. Instead of the force of gravity, centrifugal forces act.

If the rotating disk is wet by the liquid and there is no liquid supply to the center, then at the edge there is formed a stationary liquid torus. The tangent forces acting on this torus produced by air resistance can only produce liquid motion within the torus opposite to the direction of disk rotation, because of the axial symmetry of the problem. In the first approximation we will neglect this motion. Then Eqs. (13)-(18) are applicable to the rotating disk with $z = r\omega^2$, $l = 2\pi r$, i.e., the critical flow rate is determined by

$$Q_c = 4.8K_1 \frac{2\pi r}{\lambda}\left(\frac{a^3\sigma}{\rho}\right)^{1/2} \tag{19}$$

for $a = (2\sigma/\pi\rho r\omega_2)^{1/2}$ and by

$$Q_c = \frac{\pi^2}{3} K_2 \frac{r\omega}{\lambda}\left(\frac{rd^5}{2}\right)^{1/2} \tag{20}$$

at $d = (\sigma/\omega)(\sigma/\rho\, r)^{1/2}$.

An experimental verification of the results obtained was performed. For this purpose the experiments of [2] were used, in which critical flow rates Q_* were obtained for a vertical capillary for transition from drop formation (Abtropfen) to stream flow with subsequent decay of the stream into droplets (Zertropfen), as well as experiments in determining Q_* for a vertical capillary, for a rotating drum with radial orifice, for a rotating disk, and for an inclined plane with horizontal lower edge.

The physical characteristics of the liquids used in the experiments are presented in Table 1.

The rotating-drum experiments (Fig. 2a) were performed with an experimental apparatus shown schematically in Fig. 2. Into the center of drum 1 with one radial orifice 2 (or disk), rotated by electric motor 3, a stream of liquid is supplied from injection needle 4, whose control rod is loaded by weight 5. Using an ST-5 strobotachometer visual observations of the outflow of liquid from the capillary were made and the critical flow rate Q_* at which drop flow was replaced by stream flow was determined.

TABLE 2

Drop generator	Liquid	Capillary radius R, cm	Disk/drum radius r, cm	Angular velocity ω, sec^{-1}	Q cm^3/sec	K_1	K_2
Vertical capillary	Water	0.0212^1	—	—	0.103	0.624	0.244
		0.035^1	—	—	0.186	0.532	0.291
		0.056^1	—	—	0.274	0.704	0.287
		0.100^1	—	—	0.720	0.467	0.466
		0.0287	—	—	0.137	0.527	0.251
		0.0427	—	—	0.210	0.447	0.264
	Glucose solution*	0.035	—	—	0.179	0.470	0.241
		0.056	—	—	0.320	0.416	0.291
		0.100	—	—	0.656	0.370	0.346
	Mercury*	0.0085	—	—	0.0147	0.504	0.138
		0.0212	—	—	0.0444	0.389	0.193
		0.035	—	—	0.0894	0.370	0.268
	Lubricating oil	0.0287	—	—	0.054	0.312	0.192
		0.0427	—	—	0.091	0.293	0.232
	Glycerine	0.0427	—	—	0.081	0.210	0.150
	Diesel fuel	0.0287	—	—	0.113	0.635	0.391
		0.0427	—	—	0.153	0.474	0.380
Rotating drum with one orifice	Water	0.025	2.5	157	0.105	0.606	0.845
		0.025	3.5	73	0.090	0.452	0.489
		0.025	5.0	73	0.086	0.440	0.526
Inclined plane	Mineral oil	—	—	—	2.20	0.308	0.216
Rotating disk	Lubricating oil	—	—	—	1.20	0.217	0.150
	Transformer oil	—	3.5	627	0.670	0.213	0.153
		—	3.5	314	1.00	0.206	0.143
	Lubricating oil	—	3.5	157	2.00	0.272	0.194
		—	3.5	314	0.202	0.116	0.077

* Experiments of [2].

Rotating-disk experiments (Fig. 2b) were performed with the same experimental apparatus (Fig. 2), but the transition from formation of "basic" drops at the disk edge (first spray mode) to throwoff of a stream which decays into finer "secondary" drops was determined from the distribution of drop dimensions; the critical flow rate Q_* was taken as that at which basic drops disappeared and only secondary drops were formed [3].

In the inclined plane experiments the transition from drop formation to stream flow and the distance λ between adjacent drop- and stream-formation points were determined visually.

The experimental results are presented in Table 2. From the table it is evident that for the large parameter range studied with four different methods of drop formation the value of K_1, describing the first method of Q_c determination, varies from 0.116 to 0.704. The mean value is $K_1 = 0.406$, with mean-square deviation $\sigma = 0.148$ (36.4%).

For water-drop flow from a vertical capillary a decrease in drop diameter d with increase in flow rate Q occurred. This decrease did not continue to the limit $d = 2R$; transition to stream flow occurred at $d > 2R$ ($K_1 < 1$). In the case of a very viscous liquid (lubricating oil) the transition to stream flow occurred with no noticable reduction in d.

For an increase in viscosity η from 0.01 to 14.95 g/cm · sec, i.e., a factor of 1500, the coefficient K_1 varies by a factor of 2.5. For the parameter range studied the empirical function

$$K_1 = 0.53 - 0.1\,(2 - \lg \eta) \tag{21}$$

may be used.

In the second method of Q_c determination the value of K_2 varied from 0.077 to 0.845. The mean value $K_2 = 0.284$, $\sigma = 0.114$ (40.3%).

Thus, both methods of determining critical flow rate Q_c corresponding to the transition from drop flow to stream flow gave results close to experimental. The first method agrees better with available data and is to be preferred.

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